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The production spectrum of charged cosmic ray pions and the primary cosmic ray nucleon spectrum

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Abstract. The production spectrum of charged pions and the primary cosmic ray nucleon spectrum have been derived from the experimental muon spectrum determined near the geomagnetic equator at the zenith angles 45°W and 60°W. The derived pion spectrum follows the relation $P(E_\pi)\mathrm{d}E_\pi = (0{\cdot}22 \pm 0{\cdot}03)E_\pi^{-(2{\cdot}65\pm0{\cdot}12)}\,\mathrm{d}E_\pi$ and the primary spectrum agrees with the relation $N(E_\mathrm{p})\,\mathrm{d}E_\mathrm{p} = 2{\cdot}9E_\mathrm{p}^{-2{\cdot}65}\,\mathrm{d}E_\mathrm{p}$. The results have been compared with the values expected from the vertical muon spectrum.

1. Introduction

Cosmic ray primary particles comprised predominantly of charged particles (mainly protons), penetrate into the earth's atmosphere, collide with air nuclei, and produce secondary components within a depth of $100\ \mathrm{g\ cm^{-2}}$ of air. The secondary particles are predominantly pions. These pions either interact with air nuclei or decay into longer lived muons. A study of the pion production and the primary spectrum deduced from measured inclined muon spectra is necessary for an investigation into the propagation of cosmic particles in the atmosphere.

Several studies have been made of the derivation of the pion production spectrum from vertical sea-level muon spectrum measurements. These studies have been reviewed in our earlier work (Bhattacharyya 1972). Judge and Nash (1965) have derived the pion production spectrum from the inclined Nottingham muon spectra. It is evident from their investigation that the assumption of complete kaon parentage of the muon led to a far less satisfactory fit than the assumption of 100% pion parentage, indicating that at zenith angles in the range 30°–60° and below 50 GeV energies the sea-level muons are predominantly derived from pions. Moreover, the reduction of inclined sea-level muon intensity at low energy arises from the large ionization loss and high decay probability in traversing the long atmospheric path along the inclined direction.

In the present paper we report the production spectrum of pions assuming that they are all produced at an atmospheric depth of $90\ \mathrm{g\ cm^{-2}}$ in the interval 3–20 GeV, and also the corresponding primary nucleon spectrum. Our experimental sea-level muon spectra (Bhattacharyya 1974) at different zenith angles have been used. The diffusion equation for pions in the earth's atmosphere has been used. This yields the pion intensity and accounts for the loss of pions due to absorption and decay in flight. We used a model similar to that of Barrett *et al* (1952) along with some modifications. After evaluation of the pion production spectrum from the sea-level muon spectra the scaling hypothesis of Feynman (1969) has been applied to obtain the nucleon spectrum at the top of the atmosphere.

2. The muon spectra

The flash-tube range spectrograph has been operated at zenith angles 0°, 45° W and 60° W to obtain the muon spectra in the energy range 0·3–3 GeV. The details of these experiments can be found in Bhattacharyya (1970, 1974). Figure 1 shows the sea-level muon spectra at different zenith angles.

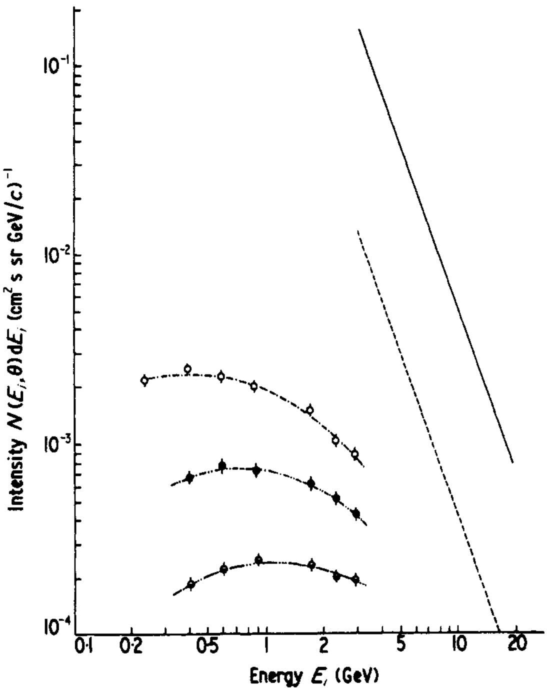

Figure 1. The differential spectra of sea-level muons at zenith angles $\theta = 0°$ (○), 45°W (●) and 60°W (◓) along with the derived spectra of pions (broken curve) and primary nucleons (full curve).

3. Procedure

3.1. Pion production spectrum from the sea-level muon spectrum

The pion production spectrum is calculated from the sea-level muon spectrum assuming that pions are the only source of muons. The pion atmospheric diffusion equation given by Barret *et al* (1952) gives the differential inclined muon spectrum $N(E, \theta)\,\mathrm{d}E$ at production for equal attenuation lengths of pions and protons in the atmosphere:

$$N(E, \theta)\,\mathrm{d}E = \frac{P(E/r)}{r}\,\frac{B_\pi}{E\cos\theta + B_\pi}\,\mathrm{d}E \tag{1}$$

where $P(E/r)$ is the differential pion energy spectrum at production; the energy degradation factor $r = m_\mu/m_\pi = 0{\cdot}76$; $B_\pi = rm_\pi c^2 H/(ct_\pi) = 90$ GeV; $m_\pi c^2 = 139{\cdot}6$ MeV; the scale height of the atmosphere $H = 6{\cdot}46$ km; the pion lifetime $t_\pi = 26$ ns and θ is the zenith angle of a muon trajectory at production. This expression can be reduced to the sea-level muon spectrum after correction for the energy loss of muons in the atmosphere and loss of muons due to μ–e decay. The production spectrum of charged pions $P(E_\pi)$ in the atmosphere can be calculated from the sea-level muon spectrum $N_\mu(E_\mu, \theta)$, using the relations given below:

$$P(E_\pi)\,\mathrm{d}E_\pi = N_\mu(E_\mu, \theta)(1 + rE_\pi \cos\theta/B_\pi) r S(E_\mu, \theta)\,\mathrm{d}E_\pi, \qquad (2)$$

where $E_\pi = E/r = 1{\cdot}32[E_\mu + E'(y_0 - y)\sec\theta]$, E' is the average energy loss for muons in the air per g cm^{-2} and is taken as 2·2 MeV after Cousins and Nash (1962); $S(E_\mu, \theta)$ accounts for the loss of muons due to decay and absorption in the atmosphere. This function is the reciprocal function of the survival probability that muons produced at a vertical depth y reach sea level depth y_0 with energy E_μ. This survival probability of muons has been calculated by using the relation taken from Rossi (1952) and the expression comes out to be of the form:

$$P_\mu(y, \theta, E_\mu) = \left(\frac{y}{y_0}\,\frac{E_\mu}{E_\mu + (y_0 - y)\sec\theta E'}\right)^{\alpha}$$

where $\alpha = B_\mu \sec\theta/(E_\mu + y_0 \sec\theta E')$; $B_\mu = m_\mu c^2 H/ct_\mu = 1{\cdot}1$; the muon lifetime $t_\mu = 2{\cdot}2$ μs; the velocity of light $c = 3 \times 10^{10}$ cm s^{-1}. Figure 2 shows the survival probability of muons produced at depth $\langle y \sec\theta \rangle_{\mathrm{av}} = 90$ g cm^{-2} reaching sea level at a depth $y_0 = 1033$ g cm^{-2} for zenith angles $\theta = 0°$, 45° and 60°, as a function of sea-level muon energy. It is evident from the plot that the probability decreases with the increase of zenith angle in the muon energy range 0·4–3 GeV. The calculated pion intensity at an atmospheric depth of about 90 g cm^{-2} has been plotted in the figure 1.

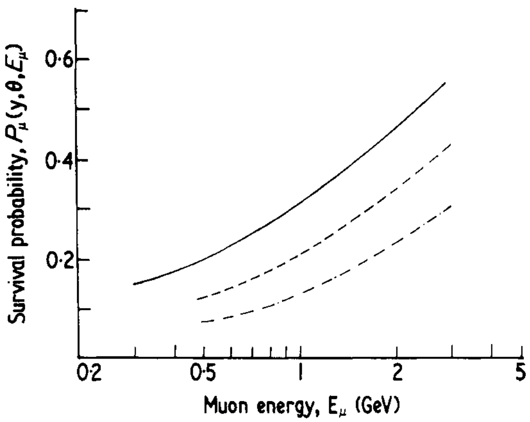

Figure 2. Survival probability of muons as functions of sea-level muon energy at different zenith angles: full curve at $\theta = 0°$; broken curve at $\theta = 45°$ and chain curve at $\theta = 60°$.

3.2. Primary nucleon spectrum from the pion production spectrum

These pions are assumed to be generated by the collisions of primary cosmic ray particles with air nuclei. The scaling hypothesis of Feynman (1969) has been used to determine the energy spectrum of primary particles at the top of the atmosphere. The detail of the procedure has already been described in Bhattacharyya (1972). The calculated primary nucleon spectrum has been plotted in figure 1.

4. Discussion

The derived spectrum of pions in the present paper is presented in table 1 along with that determined from inclined muon spectra by different authors. The spectrum of pions can be expressed by the relation $P(E_\pi)\,\mathrm{d}E_\pi = A_\pi E_\pi^{-\gamma_\pi}\,\mathrm{d}E_\pi$. The values of A_π and γ_π found by different authors are shown in table 1.

Table 1.

Authors	A_π	γ_π
Pak *et al* (1961)	0·156	2·64
Allen and Apostolakis (1961)	0·299	2·84
Ashton and Wolfendale (1963)	0·20	2·70
Judge and Nash (1965)	0·373 ± 0·04	2·97 ± 0·14
Present work	0·22 ± 0·04	2·65 ± 0·16

The sea-level muon spectra (Bhattacharyya 1970, 1974) in the spectral region 0·4–3 GeV, used to derive the pion production spectrum, was determined at low latitude (12° N geomagnetic latitude). The loss of muon intensity due to atmospheric scattering which was found to be 5–0 % in the range 0·5–1 GeV for $\theta = 45°$, and 8–0 % in the range 0·5–2 GeV for $\theta = 60°$, was neglected. Thus the derived pion production spectrum is concerned with low latitude and specified for the low energy region. We used the average value of the energy loss of muons in the low energy region (below 5 GeV) due to ionization loss in the atmosphere from Cousins and Nash (1962). The mean E' value is reasonable for the energy loss of low energy muons throughout the atmosphere and is in agreement with that expected from the results of Serre (1967). The pion production spectra in table 1 determined by other authors are mainly valid in the high energy region, namely, above 10 GeV but the present result stands for low energy pion intensity of energy above 3 GeV. Our pion spectrum determined from the inclined muon spectrum agrees well with that determined from our vertical muon spectrum. The present work agrees with the pion production spectrum determined by Ashton and Wolfendale (1963).

The scaling hypothesis of Feynman (1969) has been used to calculate the primary nucleon spectrum from the corresponding pion spectrum. The result is plotted in figure 1 and is found to follow a power law. The primary spectrum derived in the present work and the results of different authors can be presented in the form $N(E_\mathrm{p})\,\mathrm{d}E_\mathrm{p} = A_\mathrm{p}E_\mathrm{p}^{-\gamma_\mathrm{p}}\,\mathrm{d}E_\mathrm{p}$. Table 2 shows the values A_p and γ_p found by different authors along with the present work.

Table 2.

Authors	A_p	γ_p
Rayan *et al* (1972) (experimental)	2·0 ± 0·2	2·75 ± 0·03
Pal (1967)	2·37	2·65
Present work	2·90	2·65

The present result lies above the experimental spectrum of Rayan *et al* and is consistent with the work of Pal within the limits of statistical fluctuations which are about 25% (not shown in figure 1).

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